## Belleville Washers - Engineering Data

## Belleville Washer Stacking

Multiple Belleville washers may be stacked to modify the spring constant or amount of deflection. Stacking in the same direction will add the spring constant in parallel, creating a stiffer joint (with the same deflection). Stacking in an alternating direction is the same as adding springs in series, resulting in a lower spring constant and greater deflection. Mixing and matching directions allow a specific spring constant and deflection capacity to be designed.

Example: 1 Spring is considered to be 1 in Parallel, 1 in Series. (This notation is needed for load calculations)

If $\mathrm{n}=$ \# of springs in a stack, then:
Parallel Stack ( n in parallel, 1 in series) - Deflection is equal to that of one spring, Load is equal to that of $\mathrm{n} \times 1$ spring. i.e. Stack of 4 in parallel, 1 in series will have the same deflection as that of one spring and the load will be 4 times higher than that of one spring.

Series Stack ( 1 in parallel, n in series) - Deflection is equal to $\mathrm{n} \times 1$ spring, load is equal to that of one spring. i.e. Stack of 1 in parallel, 4 in series will have the same load of one spring and the deflection will be 4 times greater.

## Performance Considerations

In a parallel stack, hysteresis (load losses) will occur due to friction between the springs. The hysteresis losses can be advantageous in some systems because of the added damping and dissipation of vibration energy. This loss due to friction can be calculated using hysteresis methods. Ideally, no more than 4 springs should be placed in parallel. If a greater load is required, then factor of safety must be increased in order to compensate for loss of load due to friction. Friction loss is not as much of an issue in series stacks

In a series stack, the deflection is not exactly proportional to the number of springs. This is because of a bottoming out effect when the springs are compressed to flat. The contact surface area increases once the spring is deflected beyond $95 \%$. This decreases the moment arm and the spring will offer a greater spring resistance. Hysteresis can be used to calculate predicted deflections in a series stack. The number of springs used in a series stack is not as much of an issue as in parallel stacks.

Belleville washers are useful for adjustments because different thicknesses can be swapped in and out and they can be configured differently to achieve essentially infinite tunability of spring rate while only filling up a small part of the technician's tool box. They are ideal in situations where a heavy spring force is required with minimal free length and compression before reaching solid height. The downside, though, is weight, and they are severely travel limited compared to a conventional coil spring when free length is not an issue.

A similar device is a wave washer.

## Calculation Example

If friction and bottoming-out effects are ignored, the spring rate of a stack of identical Belleville washers can be quickly approximated. Counting from one end of the stack, group by the number of adjacent washers in parallel. For example, in the stack of washers in Figure 1, the grouping is 2-3-1-2, because there is a group of 2 washers in parallel, then a group of 3 , then a single washer, then another group of 2 .

Figure 1:


The total spring coefficient is:
$K=\frac{k}{\sum_{i=1}^{g} \frac{1}{n_{i}}} \quad K=\frac{k}{\frac{1}{2}+\frac{1}{3}+\frac{1}{1}+\frac{1}{2}} \quad K=\frac{3}{7} k$
Where:
$n_{i}=$ the number of washers in the ith group
$\mathrm{g}=$ the number of groups
$\mathrm{k}=$ the spring constant of one washer
So, a 2-3-1-2 stack (or, since addition is commutative, a 3-2-2-1 stack) gives a spring constant of 3/7 that of a single washer. These same 8 washers can be arranged in a $3-3-2$ configuration ( $K=6 / 7^{*} k$ ), a 4-4 configuration ( $K=2^{*} k$ ), a 2-$2-2-2$ configuration ( $K=1 / 2^{*} k$ ), and various other configurations. The number of unique ways to stack $n$ washers is defined by the integer partition function $p(n)$ and increases rapidly with large $n$, allowing fine-tuning of the spring constant. However, each configuration will have a different length, requiring the use of shims in most cases.

Figure $\mathbf{2}$ provides a quick summary of Belleville Washer stacking characteristics.
Figure 2:

## Characteristics of Belleville Washer Stacks With Same Size Washers


a) Single washer.
b) Two washers stacked in parallel (double force at same deflection)
c) Spring column with three single washers stacked in series (triple deflection)
d) Spring column with three parallel pairs arranged in series (double force, triple deflection)

